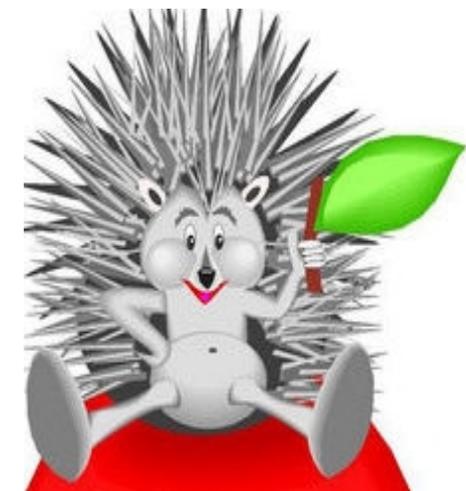


QCD topology, chiral symmetry breaking and confinement

Edward Shuryak
Stony Brook
(Lattice 2014, Columbia, June 2014)

in collaboration with
Tin Sulejmanpasic and
Pietro Faccioli



Our group's logo
emphasizing our
interest in topology

QCD topology, chiral symmetry breaking and confinement

"Instantons are not toys!" (P.van Baal)

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“magnetic scenario”: Liao, ES hep-ph/0611131, Chernodub+Zakharov

Old good Dirac condition

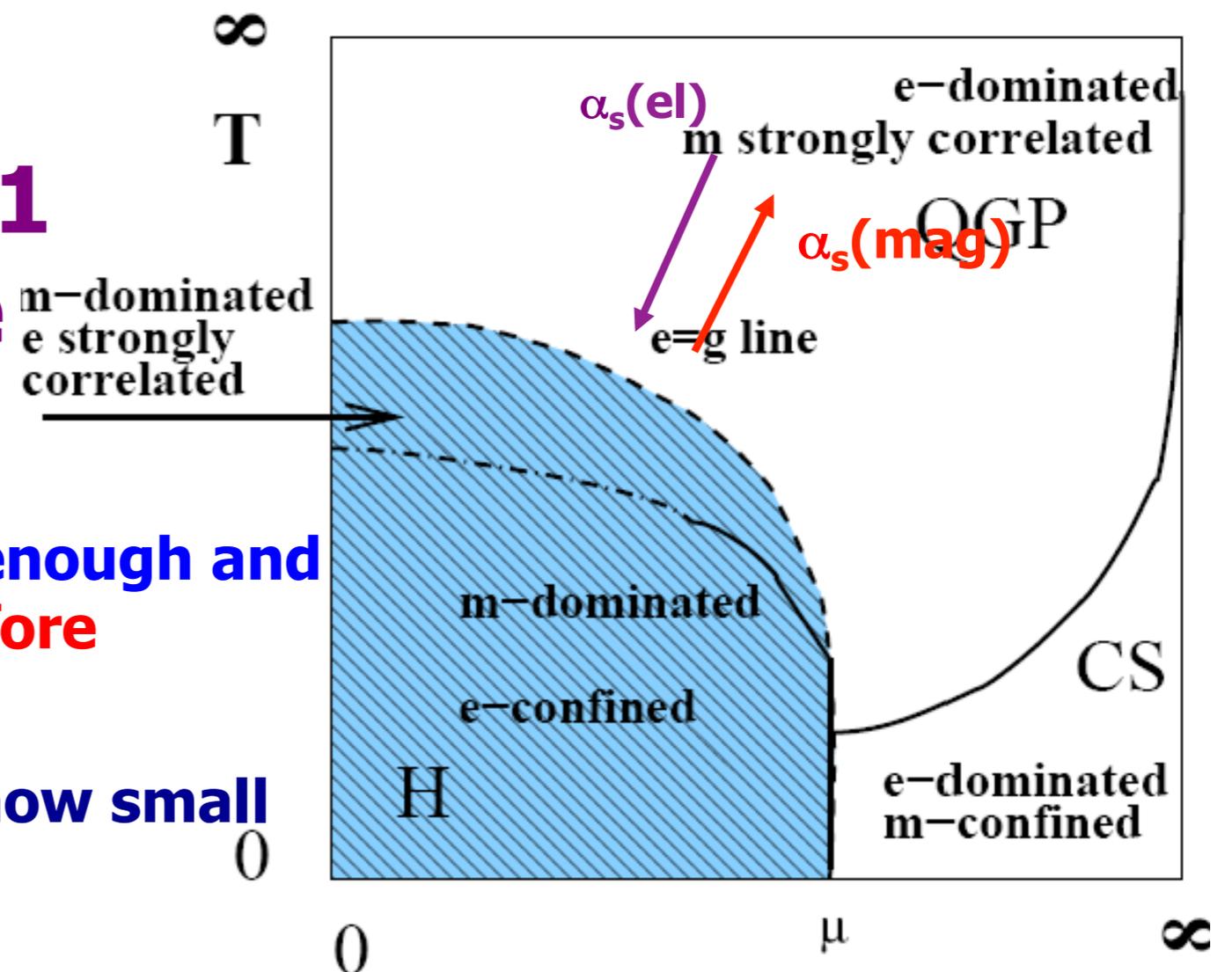
$$\alpha_s(\text{electric}) \quad \alpha_s(\text{magnetic}) = 1$$

=> electric/magnetic couplings (e/g) must run in the opposite directions!

the “equilibrium line”
 $\alpha_s(\text{el}) = \alpha_s(\text{mag}) = 1$
 needs to be in the plasma phase

monopoles should be dense enough and sufficiently weakly coupled before deconfinement to get BEC

=> $\alpha_s(\text{mag}) < \alpha_s(\text{el})$: how small can $\alpha_s(\text{mag})$ be?



terminology

- **particle-monopoles**, 3d particle-like objects with nonzero magnetic charge. Its Bose condensate makes “dual superconductor” and confinement. *Not a solution in pure gauge, not to be discussed in this talk, though*

- **instanton-***

*=dyon (Diakonov et al, ES et al)

*= monopole (Unsal et al)

*=quark (Zhitnitsky et al)

the same
object

(anti)selfdual 3d YM solution at nonzero holonomy

with electric and magnetic charges, a constituent of the instanton. Not a particle=> no paths or condensates, Z is an integral over locations only

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! in $N=2$ SYM (Seiberg-Witten theory) when both are under control, and can prove that $\text{stat.sum } Z$ over **particle-monopoles** and

instanton dyons are equal !

(being low and high-T approaches to the same physics)

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**! monopoles were used before to understand
confinement**

**instantons were used to understand chiral breaking,
and instanton-dyons seem to be able to do **both!****

outline

- nonzero holonomy \Rightarrow instanton-dyons and their interaction, “high-T confinement”, modified Coulomb
- Chiral symmetry breaking, Zero Mode Zone and instantons,
- fermionic zero modes of the dyons, dyon-antidyon pairs
[ES and T.Sulejmanpasic, arXiv:1201.5624](#), [R.Larsen and ES, in progress](#)
- **Numerical simulations of the dyon ensemble**
[P.Faccioli+ES, archive 1301.2523 Phys. Rev. D 87, 074009 \(2013\)](#)
- **back reaction to holonomy potential \Rightarrow confinement**
[ES and T.Sulejmanpasic, arXiv:1305.0796](#), inspired by [Poppitz, Schafer and Unsal, arXiv:1212.1238](#)

holonomy and the onset of confinement

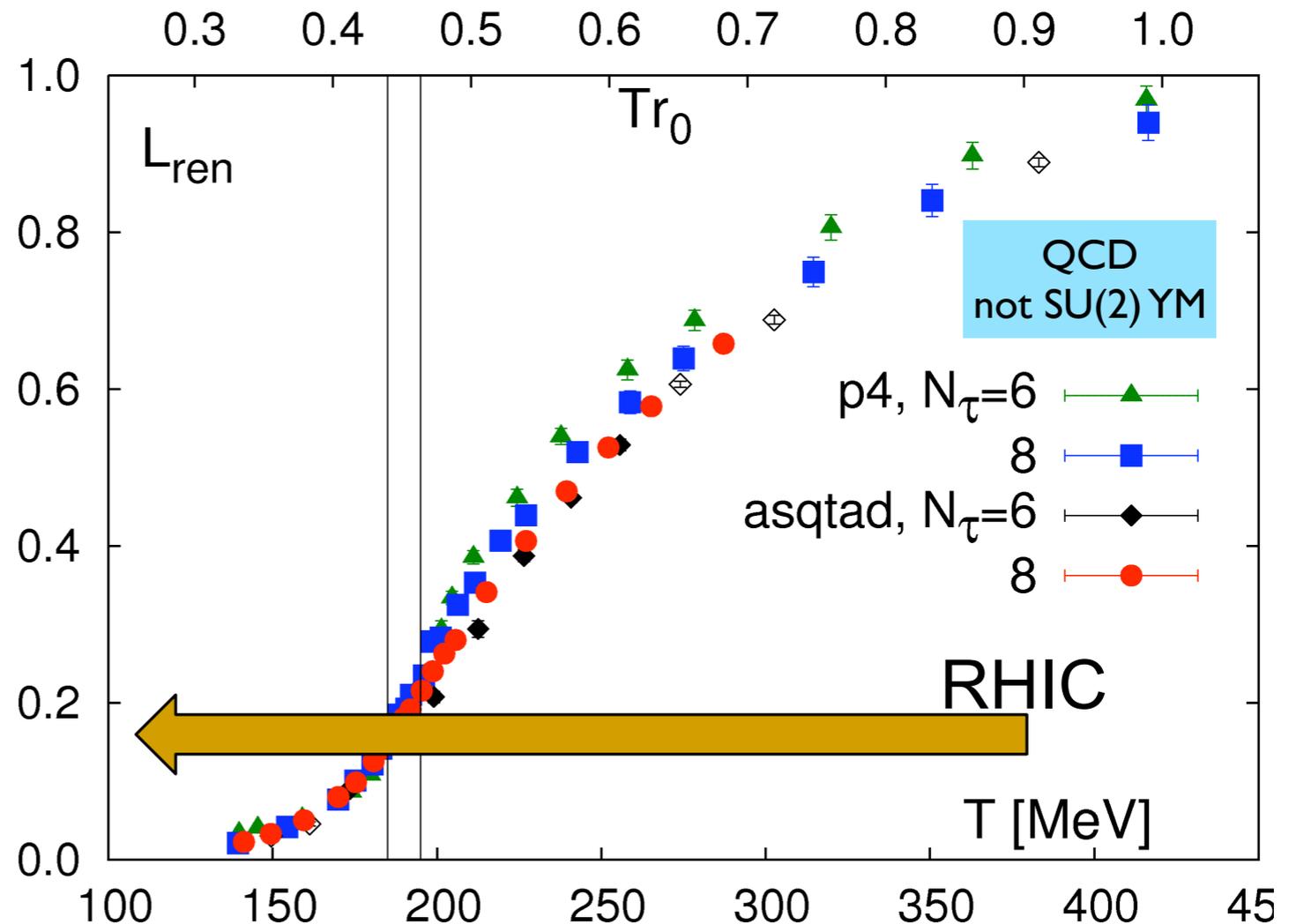
$$L = \langle P \rangle = \left\langle \frac{1}{N_c} \text{Tr} P \exp\left(i \int d\tau A_0\right) \right\rangle$$

$$= e^{-F(\text{quark})/T}$$

The Polyakov loop

$L=1 \Rightarrow A_0=0$ high T full QGP
 $L=1/2$ “**semi-QGP**” (Pisarski)
 $L \rightarrow 0$ no quarks or onset of confinement

popular models like PNJL and PSM, make semi-QGP quantitative



The approximate width of the phase transition in thermodynamical quantities, energy and entropy is small, but P changes between T_c and $2T_c$

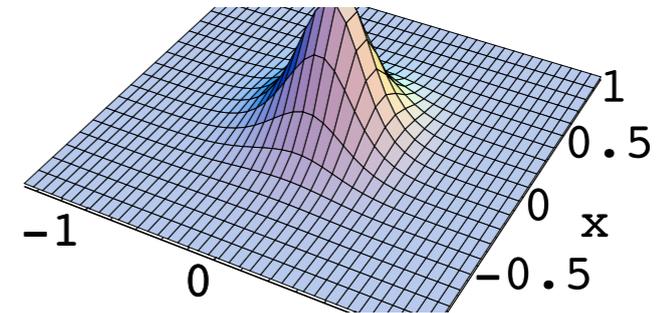
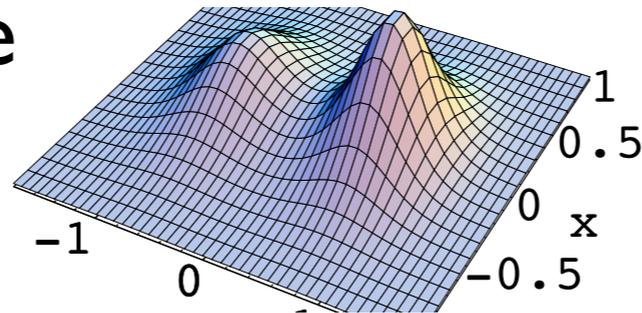
Instantons => Nc selfdual dyons

(P.van Baal et al)

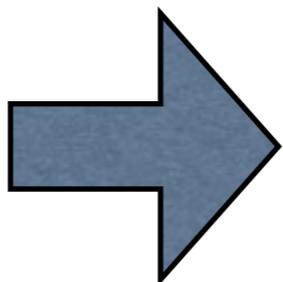
$\langle P \rangle$ nonzero Polyakov line

$\Rightarrow \langle A_4 \rangle$ nonzero

\Rightarrow new solutions



Instanton liquid
4d+short range



Dyonic plasma
3+1d long range

**instanton-
dyons in
SU(2)**

name	E	M	mass
M	+	+	v
\bar{M}	+	-	v
L	-	-	$2\pi T - v$
\bar{L}	-	+	$2\pi T - v$

calorons=M+L
are
E and M neutral

TABLE I: The charges and the mass (in units of $8\pi^2/e^2T$) for 4 SU(2) dyons.

calorons (finite-T) were located on the lattice
 Ilgenfritz et al, Gattringer...
 are instanton-dyons semiclassical?

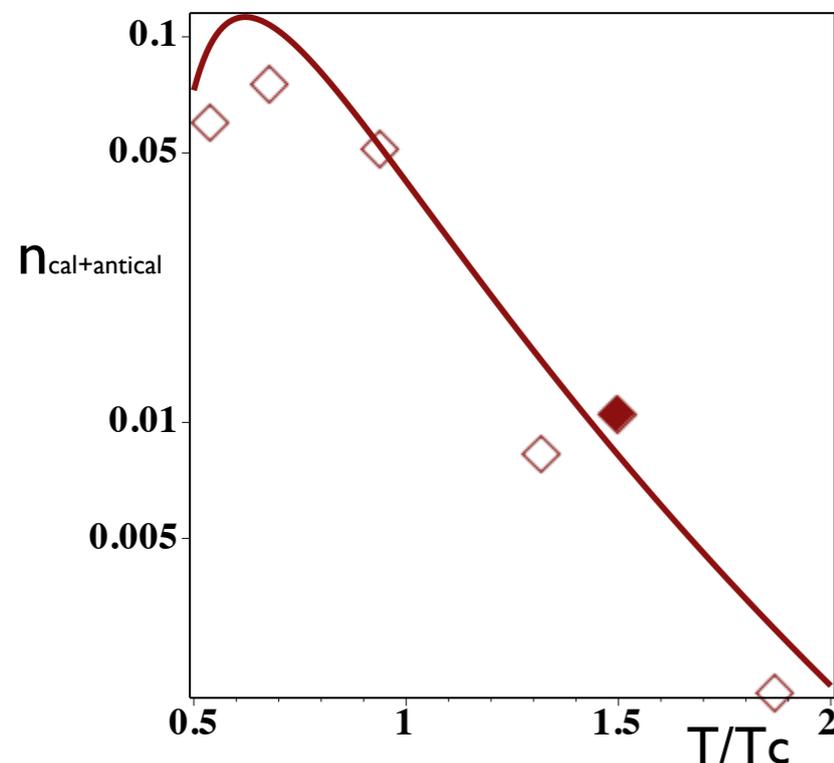


FIG. 1: Caloron density as a function of T/T_c . The solid curve is the semiclassical fit $n_{cal} = K S_{cal}^2 e^{-S_{cal}}$ in units of T with parameters $K = 0.024$, $S_{cal} = 8\pi^2/g^2(T)$, open (filled) points are the lattice data from [19] ([20]).

$$n_{cal+c\bar{a}l} = K S_{cal}^4 e^{-S_{cal}}, \quad S_{cal} = \frac{22}{3} \ln \left(\frac{T}{\Lambda} \right) \quad (1)$$

with parameters¹ $K = 0.024$, $\Lambda/T_c = .36$. The caloron action at T_c is 7.50, so per dyon it makes $S_d = S_{cal}/2 = 3.75$, which gives an idea how semiclassical the discussed objects are. (SU(3) instantons have actions $S_{cal} \approx 12$ or $S_d = S_{cal}/3 \approx 4$, quite close in magnitude.) After those parameters are fixed, one knows semiclassical densities of the dyons and their pairs, as we explain in detail below.

dimensionless density
 n/T^4

so $S_c=4 \gg 1$ or $\exp(-4)$
 is my semiclassical parameter

at $T > T_c$ M are lighter than L,
 both were identified
 on the lattice

Chiral symmetry breaking and ZMZ

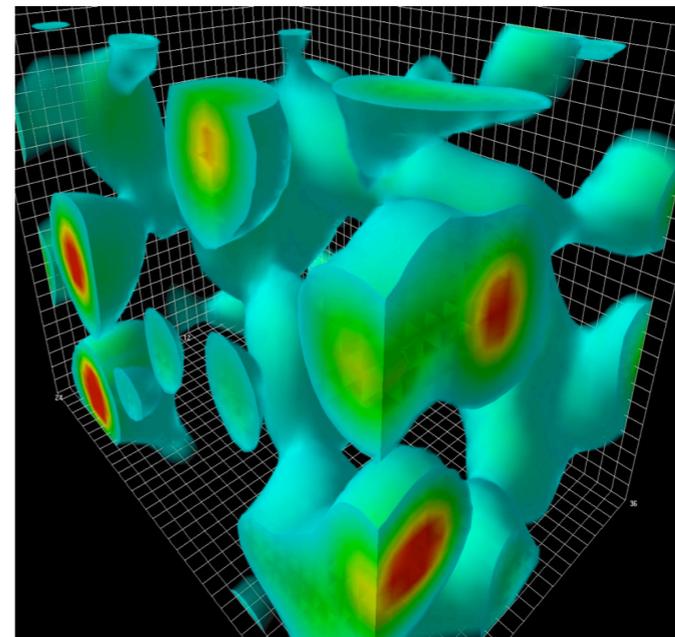
QCD vacuum is (the least expensive) topological material

$$(G_{\mu\nu}^a)^2 = \frac{192\rho^4}{(x^2 + \rho^2)^4}$$

$$A_{\mu}^a(x) = \frac{2}{g} \frac{\eta_{a\mu\nu} x_{\nu}}{x^2 + \rho^2}$$

theory and phenomenology of the instanton ensemble in the QCD vacuum

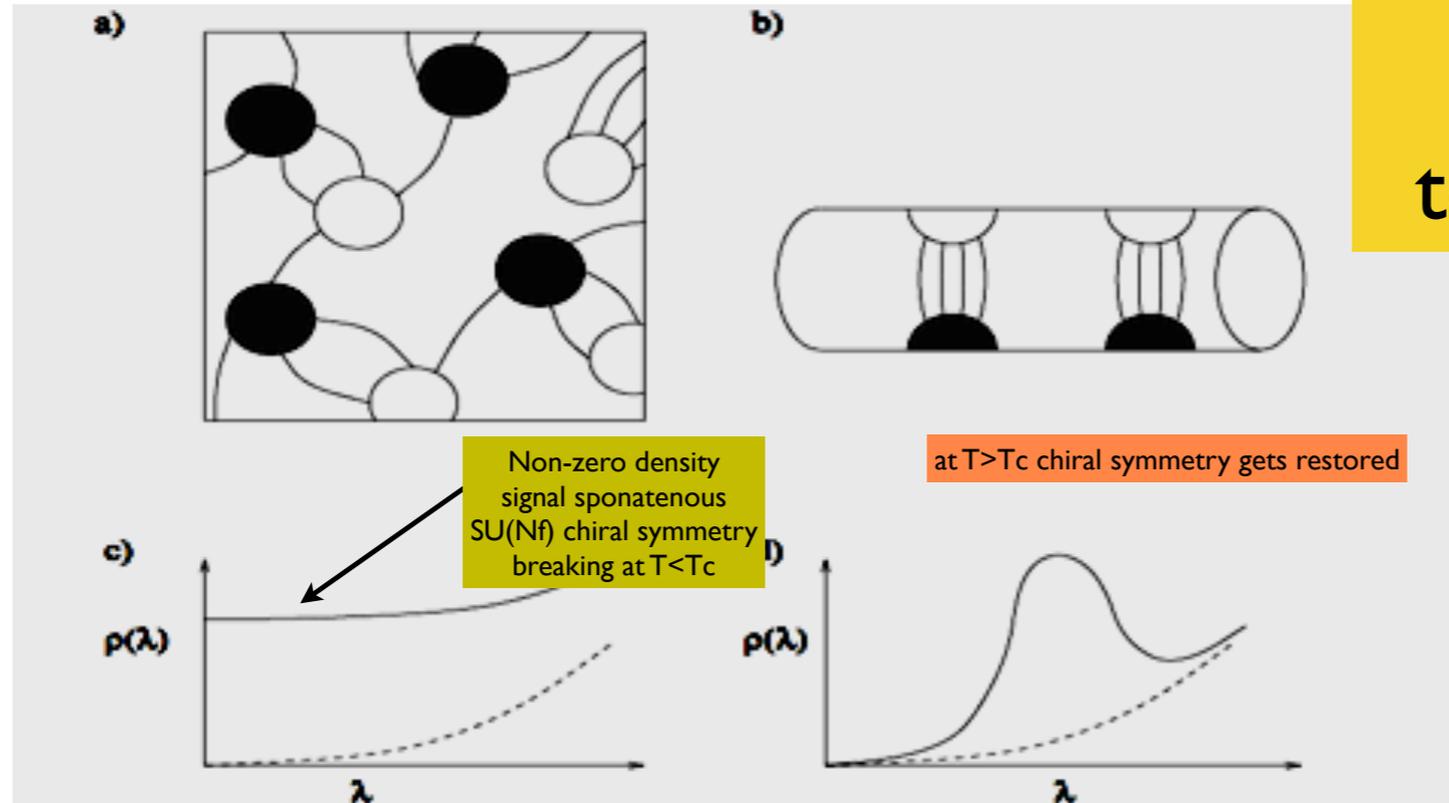
- (1982) ES: the instanton liquid model: $n=1 \text{ fm}^{-4}$, $\rho=.3 \text{ fm}$, small diluteness $n \rho^4$
- $\langle G^2 \rangle, \langle Q^2 \rangle$ and $\langle \bar{\psi}\psi \rangle$ as 3 inputs \Rightarrow 2 params +check
- 1990's IILM
- QCD vacuum and instantons
(T.Schaefer, ES, RMP 1996)



topology on the lattice
Adelaide group, 2000

Instanton liquid at $T=0$ and $T>T_c$ (schematic pictures)

fundamental concept:
ZMZ,
a collectivized set of
topological zero modes



$$D_{\mu} \gamma_{\mu} \psi_{\lambda} = \lambda \psi_{\lambda}$$

density of states (0) =>
nonzero quark condensate
``conductor'' at low T

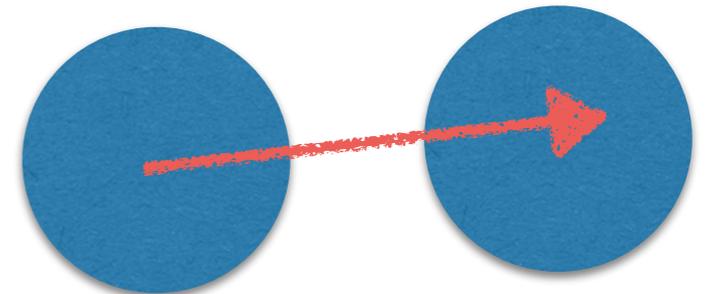
zero density of states (0) =>
zero quark condensate
``insulator'' at high T

chiral symmetry transition is thus
understood in a ``single-body'' language
as conductor-insulator transition in 4d

the width of the ZMZ is surprisingly small

the magnitude of the hopping from one instanton to the next
can be estimated as

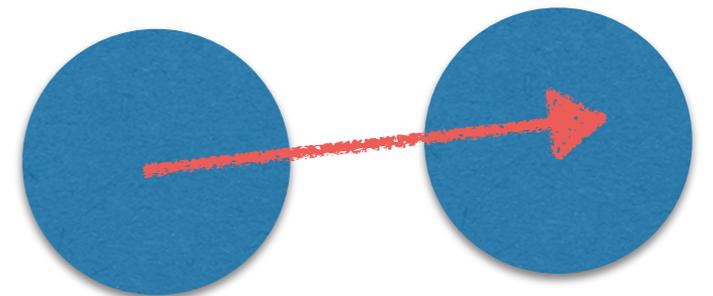
$$T_{I\bar{I}} \sim \frac{\rho^2}{R^3} \sim \frac{(0.3 \text{ fm})^2}{(1 \text{ fm})^3} \sim 20 \text{ MeV}$$



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- that is why quark mass dependence is nontrivial when m is of this order, and chiral perturbation extrapolations are not as good as people hoped!

recently the opposite exercise was done by the Graz group

Symmetries of hadrons after unbreaking the chiral symmetry

L. Ya. Glozman,^{*} C. B. Lang,[†] and M. Schröck[‡]

Institut für Physik, FB Theoretische Physik, Universität Graz, A-8010 Graz, Austria

By eliminating ZMZ strip with width σ (about 50 modes or 10^{-4} of all) one changes masses $O(1)$: near-perfect chiral pairs are left

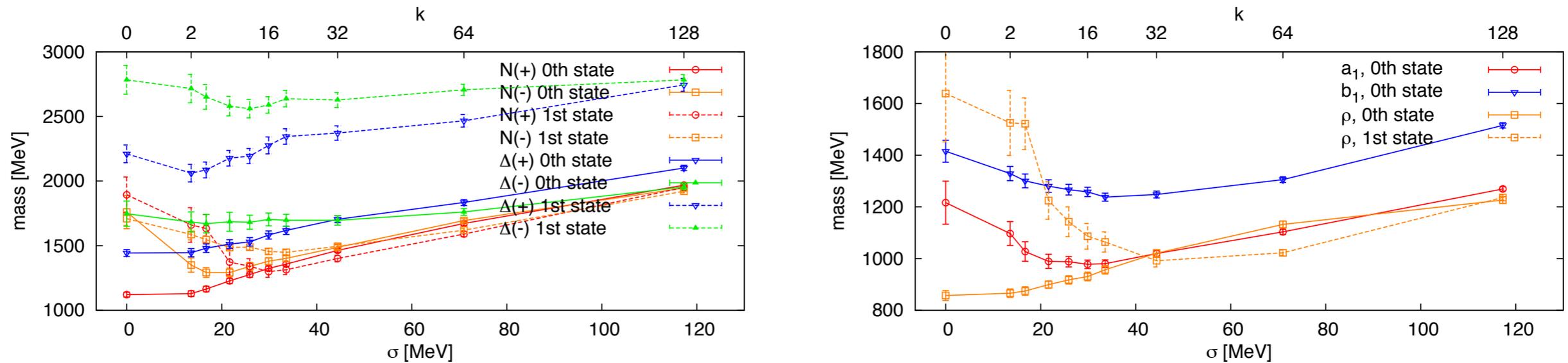


FIG. 13. Summary plots: Baryon (l.h.s.) and meson (r.h.s.) masses as a function of the truncation level.

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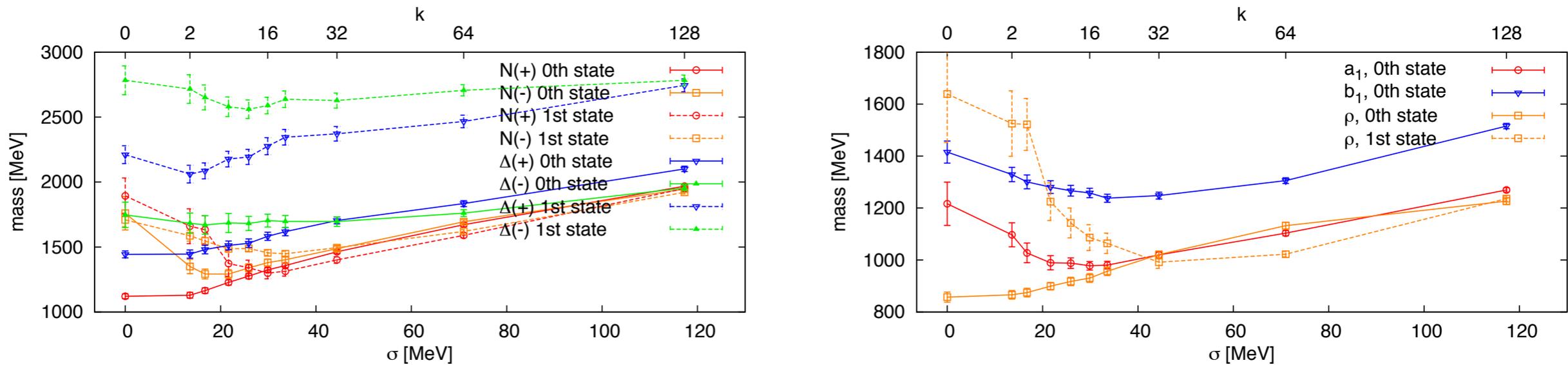


FIG. 13. Summary plots: Baryon (l.h.s.) and meson (r.h.s.) masses as a function of the truncation level.

arXiv:1205.4887v3 [hep-lat] 19 Jul 2012

Comment for lattice practitioners: the ZMZ states are also responsible for most of the statistical noise in simulations with dynamical fermions: ZMZ needs attention!

lattice puzzle

(which worried me from around 2000)

lattice puzzle

(which worried me from around 2000)

- (Gattringer et al): while quenched (pure YM) gauge ensembles show chiral restoration at $T > T_c$ for **antiperiodic** quarks,
- and yet, it is **not so for periodic quarks!**
(not physical but need to be understood anyway. One can do arbitrary periodicity angle as well, and see a gradual transition as well)
- an instanton has **one zero mode, whatever fermions one uses!**
- let me repeat, the ensemble is quenched, so no back reaction. It is the same gauge fields, and this makes the puzzle harder to solve

(going ahead of myself)

predictions: densities of the M and L dyons

crosses: “unidentified topological objects”, an upper limit

circles: identified M

L dyon size is very small and measuring $\langle P \rangle$ at its center is hard, as well as E and M charges: not done yet

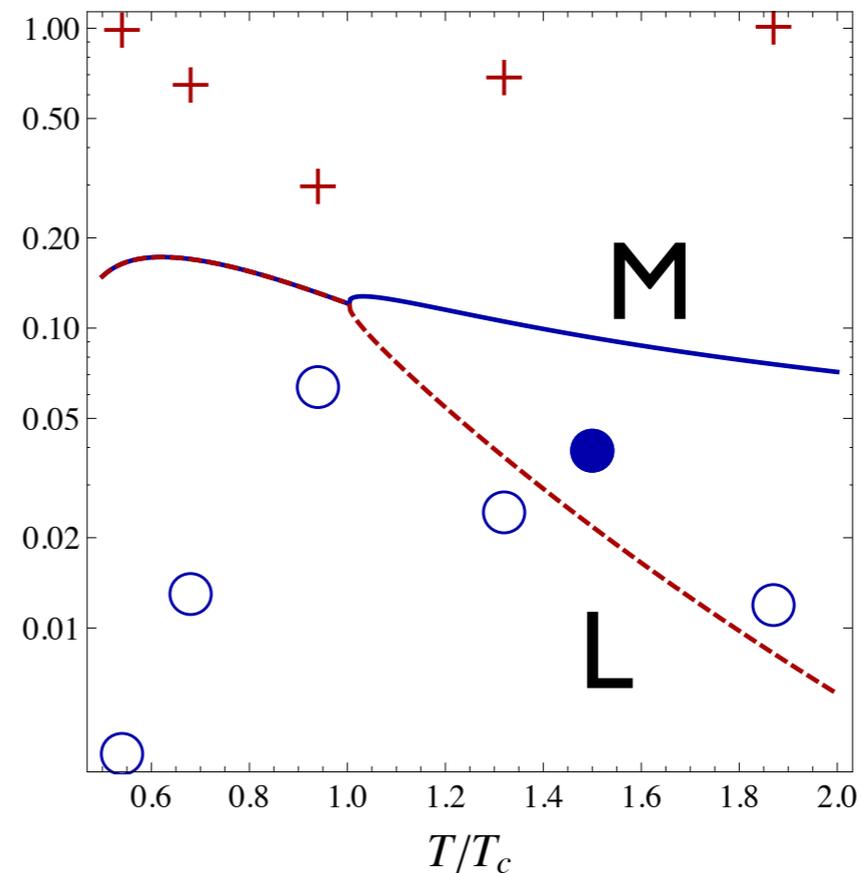


FIG. 3: Prediction of the model for the temperature dependence of the density of the instanton-dyons are shown by the lines, those with solid and dashed lines are for M, L type dyons, respectively. Open (filled) circles show identified M -type dyons from ref. [19] ([20]). The crosses show “unidentified topological objects” from [19]. Circles and crosses provide the lower and the upper bound for the dyon density.

fermionic zero modes of the instanton-dyons

- antiperiodic quarks have zero modes with L
- periodic quarks with M dyons

$$\psi_0 \sim \exp(-mr) / \sqrt{r}$$

this solves the puzzle: the density of M n_M is larger than n_L because they are lighter

If there are light dynamical (antiperiodic) quarks, they bind $L\bar{L}$ pairs, more tightly if N_f is large

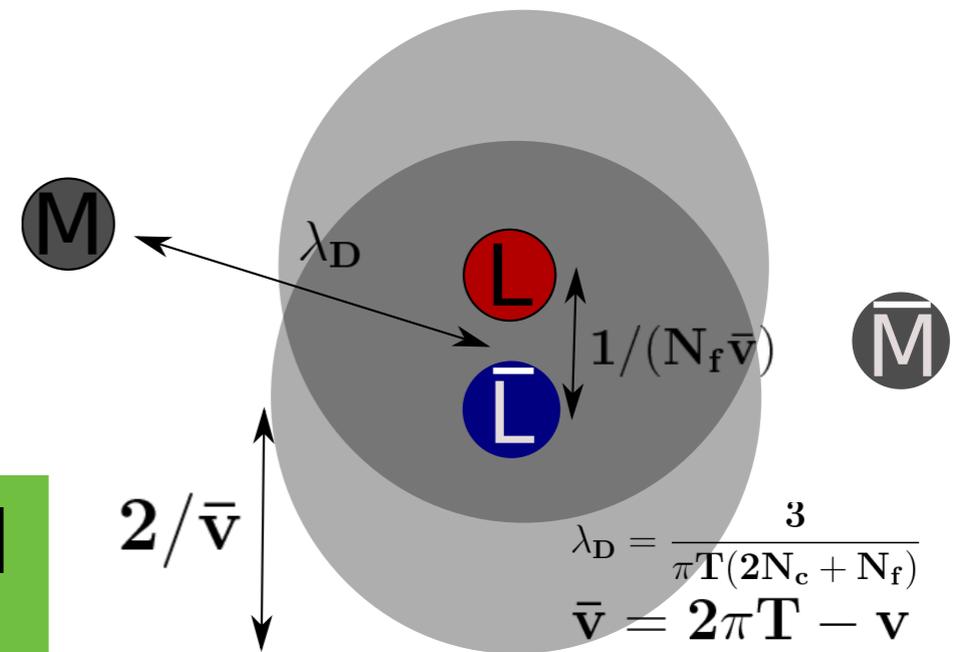
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The screening by the plasma (ES, Pisarski-Yaffe, Diakonov)

Statmech of the dyons

$$Z = \int \{dX_i\} e^{-S_c} \det G \det F_{zm} \frac{\det' F_{nzm}}{\sqrt{\det' B}}$$

The moduli space metric
(Atiyah, Hitchin, Diakonov)

in a dilute case provides
electric and magnetic
Coulomb with natural
charges

If dense produces
regularization and
repulsive core

Fermionic determinant in zero mode
approximation (ES, Sulejmanpasic), only
for L dyons

In evaluating the fermionic determinant, the Dirac operator is approximated by retaining only the contribution evaluated on the subspace of fermionic zero-modes of the individual pseudo-particles $|\phi_0^j\rangle$:

$$\text{Det}(i\gamma_\mu D^\mu + im) \simeq \text{Det}(\hat{T} + im), \quad (22)$$

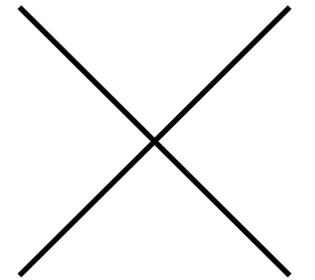
where

$$T_{ij} = \langle \phi_0^i | i\gamma_\mu D^\mu | \phi_0^j \rangle. \quad (23)$$

This scheme was well tested in the framework of the instanton liquid model, where it corresponds to summing up all loop diagrams created by 't Hooft effective Lagrangian.

“near-confinement” of the instanton-quarks (Diakonov et al)

thermal
 $p=O(T)$



$A_{4,p=0}$

In $SU(2)$: thermal quanta in QGP scatter on the instanton and generate linear potential

$$V_{12} \sim \langle (A_4)^2 \rangle = \int d^3x \left| \frac{1}{r_L} - \frac{1}{r_M} \right|^2 = 4\pi r_{LM}$$

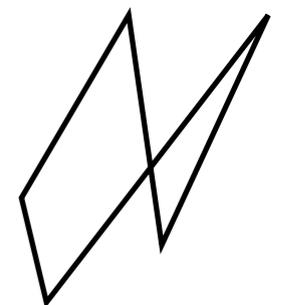
We further note that the form (14) can be obtained directly by the instanton screening term calculated by Pisarski and Yaffe [30] by recalling that the instanton size ρ and the $L - M$ separation are related by the expression

$$\pi\rho^2 T = r_{ML}. \quad (17)$$

which relates the “4-d dipole” of the instanton field to the “3-d dipole” of the dyon LM pair made of opposite charges.

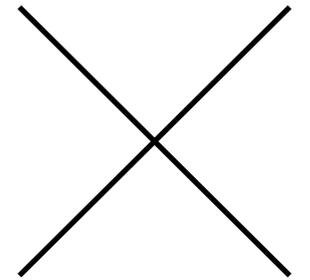
In $SU(N_c)$: instanton=baryon
linear potential \Rightarrow perimeter of a polygon

$$V \sim M_D^2 \sum_{i=1, N_c} |r_{i, i+1}|$$



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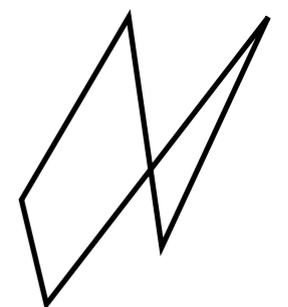
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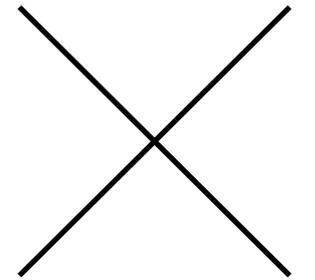
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not really a confinement as A_0 is massive,



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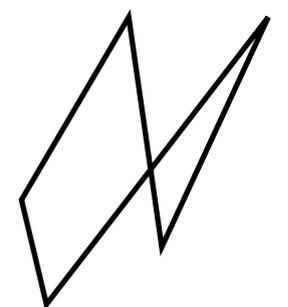
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PHYSICAL REVIEW D **87**, 074009 (2013)

QCD topology at finite temperature: Statistical mechanics of self-dual dyons

Pietro Faccioli^{1,2} and Edward Shuryak³

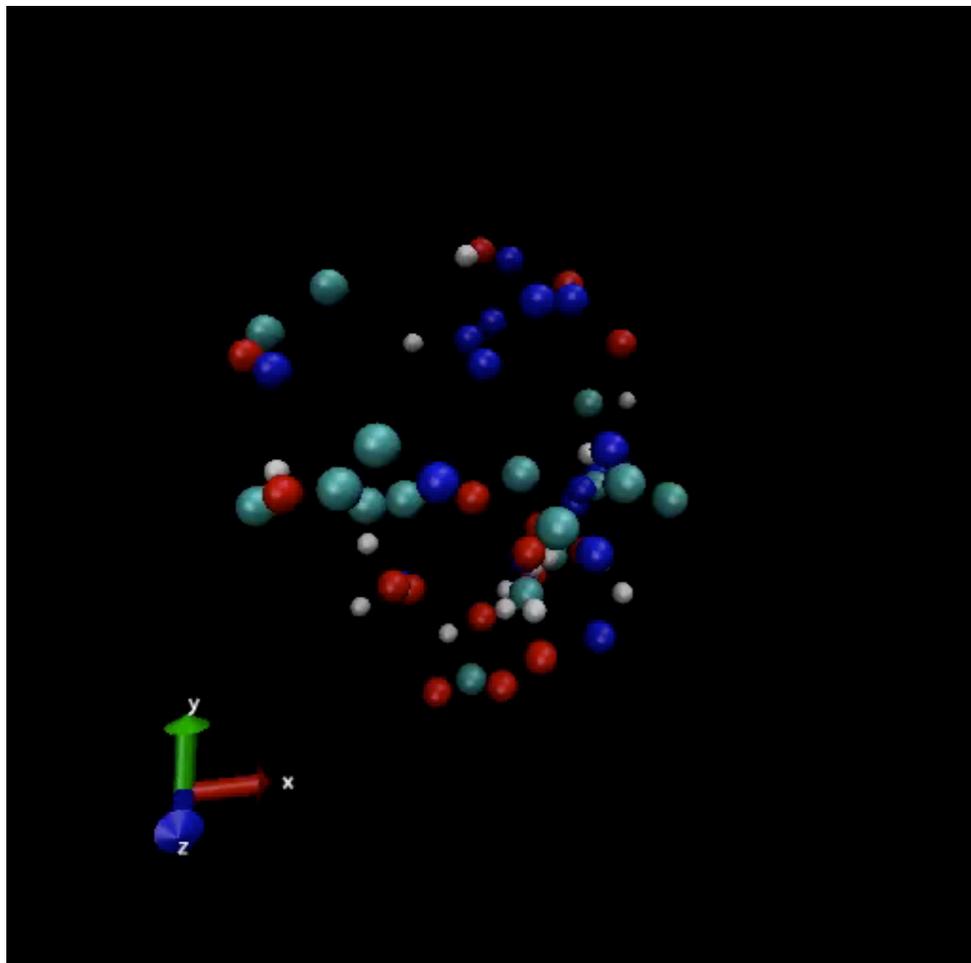
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³Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA

(Received 27 January 2013; published 9 April 2013)

Topological phenomena in gauge theories have long been recognized as the driving force for chiral symmetry breaking and confinement. These phenomena can be conveniently investigated in the semiclassical picture, in which the topological charge is entirely carried by (anti-)self-dual gauge configurations. In such an approach, it has been shown that near the critical temperature, the nonzero expectation value of the Polyakov loop (holonomy) triggers the “Higgsing” of the color group, generating the splitting of instantons into N_c self-dual dyons. A number of lattice simulations have provided some evidence for such dyons, and traced their relation with specific observables, such as the Dirac eigenvalue spectrum. In this work, we formulate a model, based on one-loop partition function and including Coulomb interaction, screening and fermion zero modes. We then perform the first numerical Monte Carlo simulations of a statistical ensemble of self-dual dyons, as a function of their density, quark mass and the number of flavors. We study different dyonic two-point correlation functions and we compute the Dirac spectrum, as a function of the ensemble diluteness and the number of quark flavors.



← density

The first statistical simulations

Coulomb ++
64 dyons on S^3 ,
Faccioli+ES

Coulomb +-
fermions

N_f



fermions

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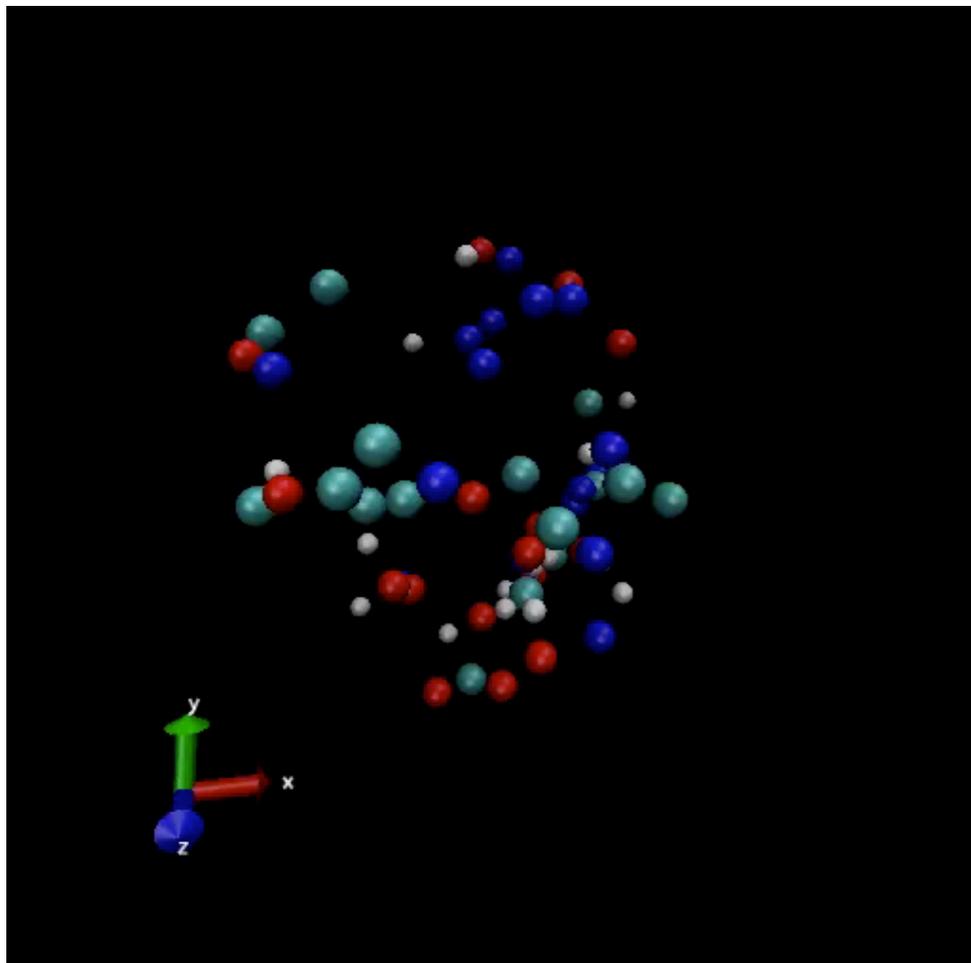
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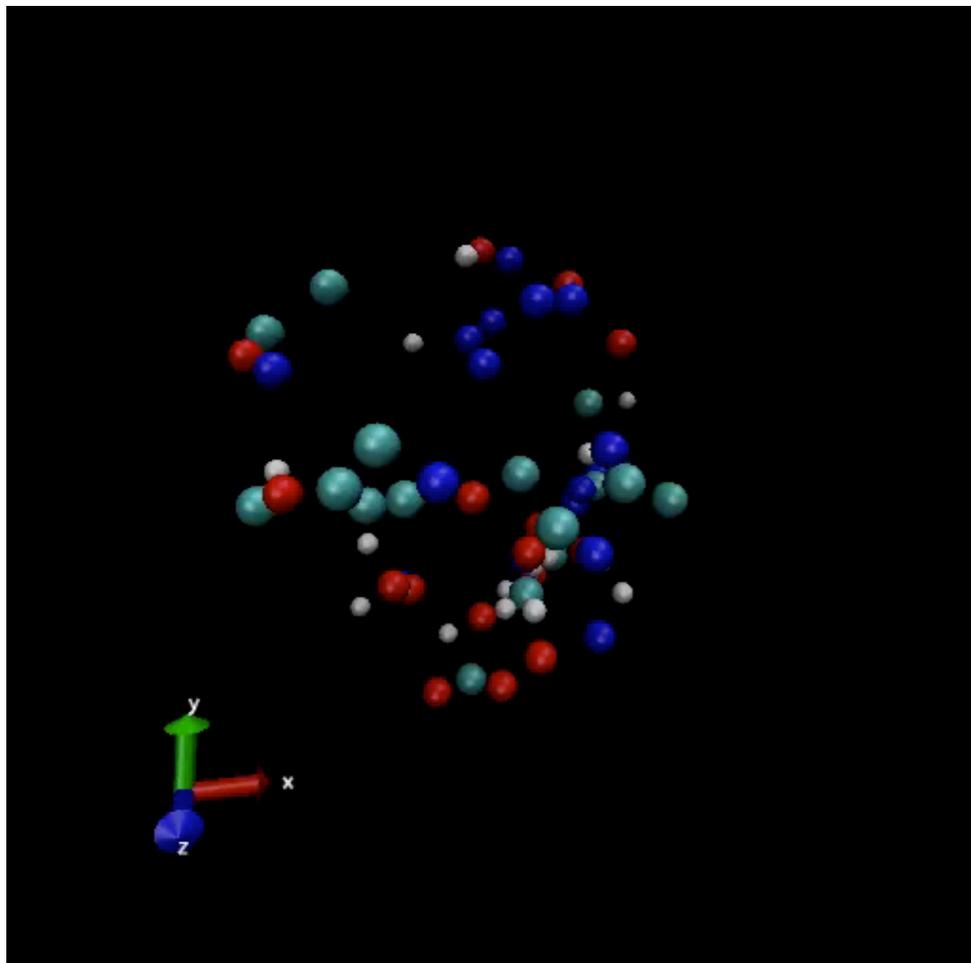
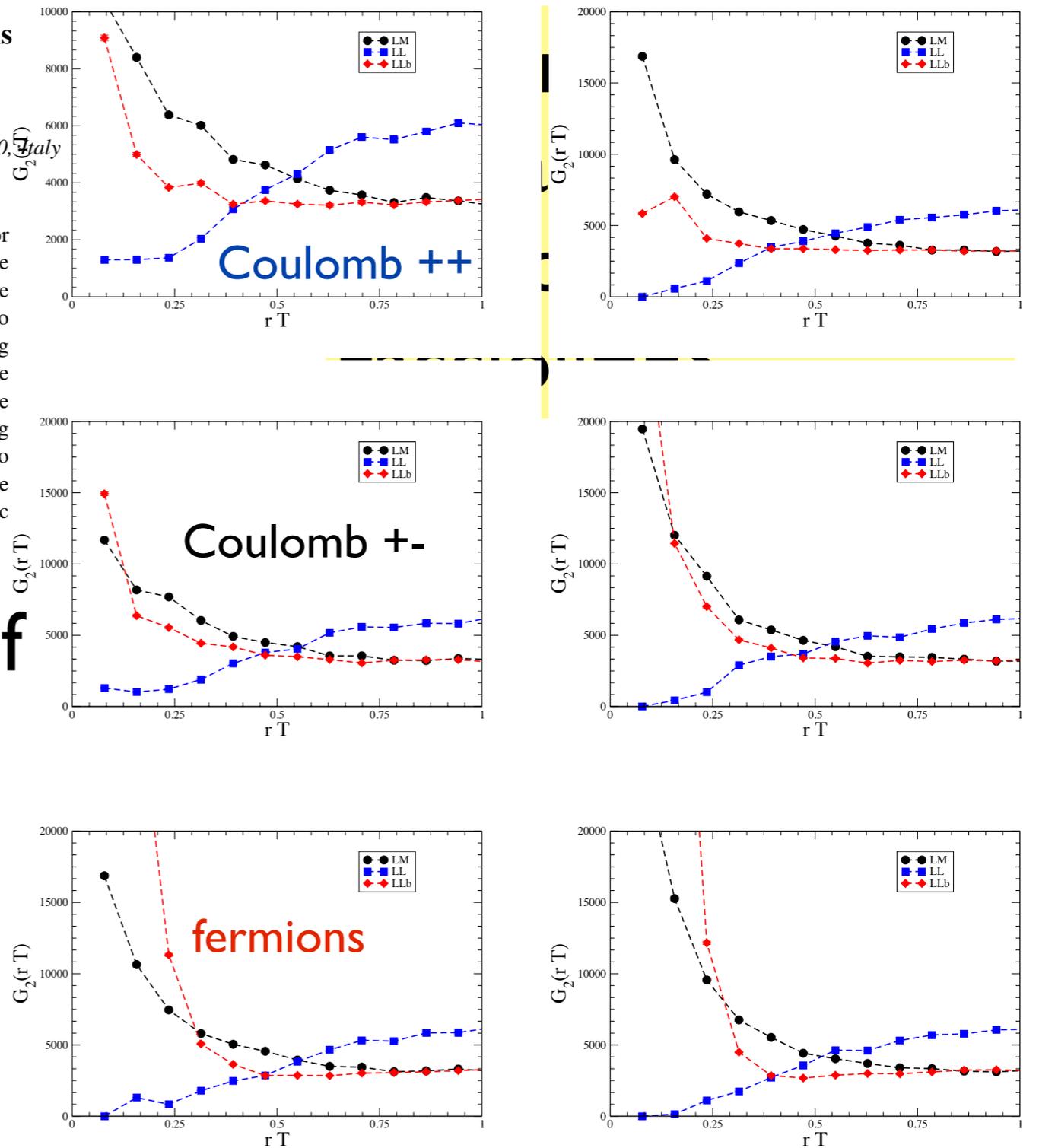
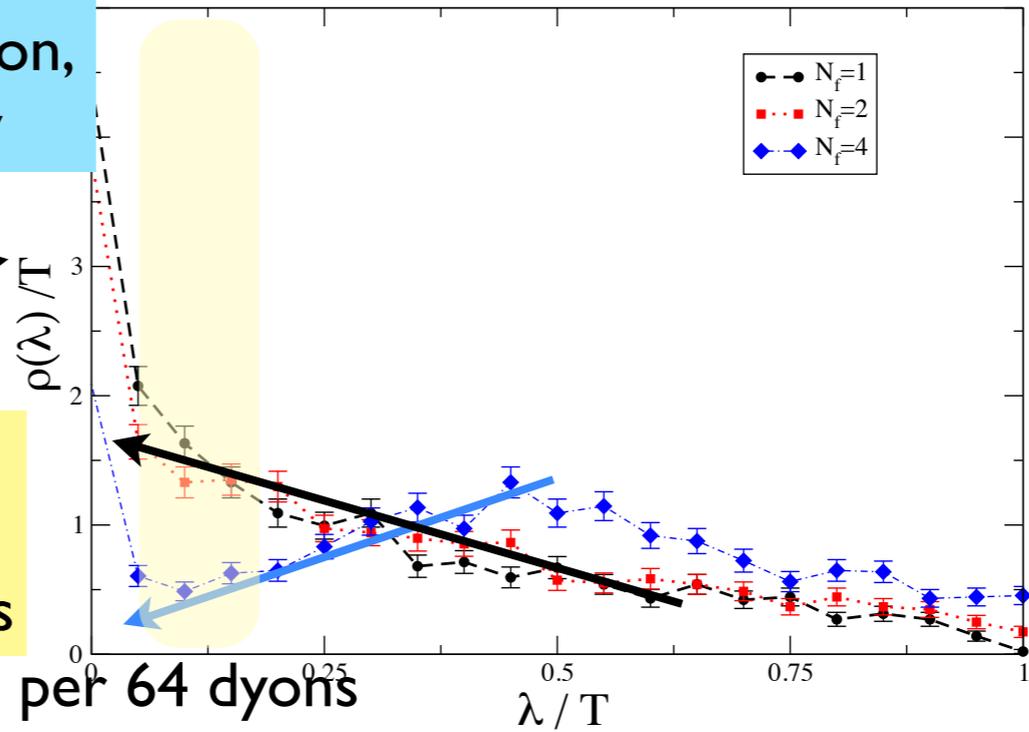

 N_f


FIG. 2: The correlation function for LM , LL and $LL\bar{L}$ dyons versus distance, normalized to the volume available. From top to bottom we show $N_f = 1, 2, 4$, respectively. Left/right columns are for the volumes per dyon $VT^3 = 0.31, 1.04$.

dyons	$R(S^3)T$	$VT^3/dyon$
64	4.5	28.
64	3.0	8.3
64	2.5	4.8
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64	1.2	0.53
64	1.	0.31

Example of the Dirac eigenvalue distribution, high dyon density

at small Dirac eigenvalues one finds known finite volume effects

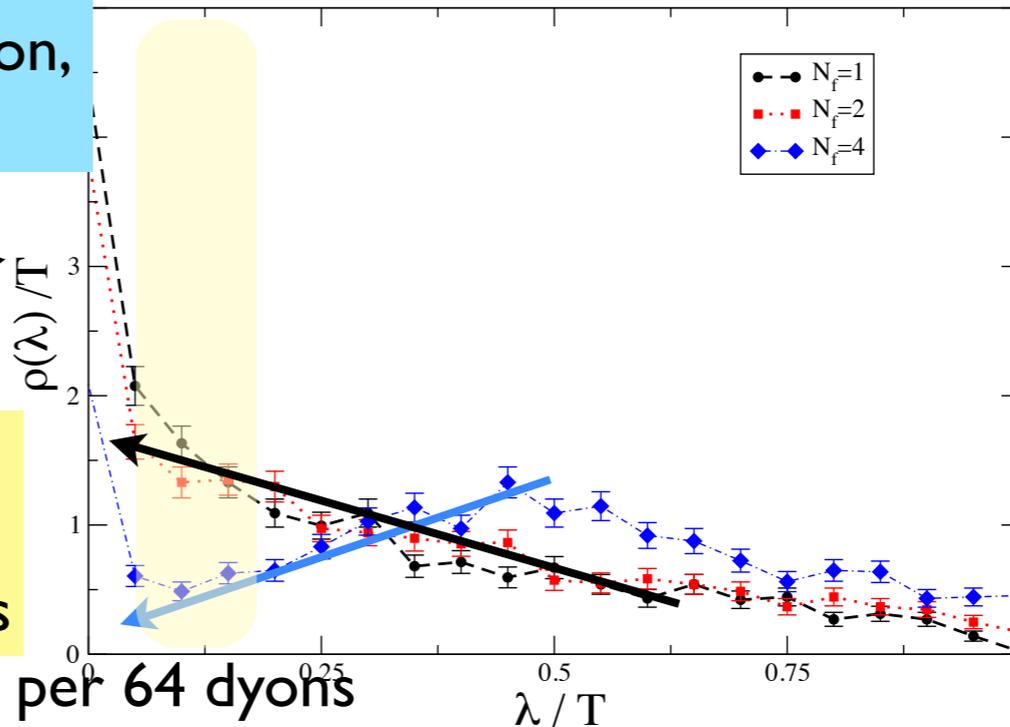


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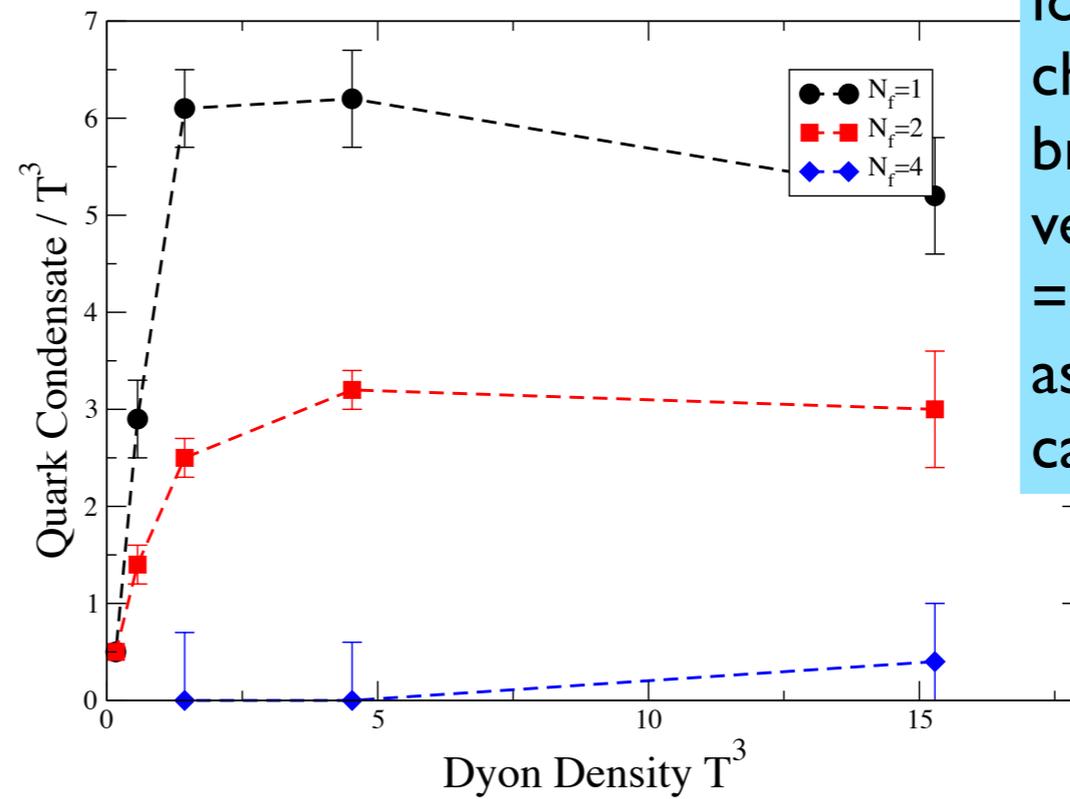
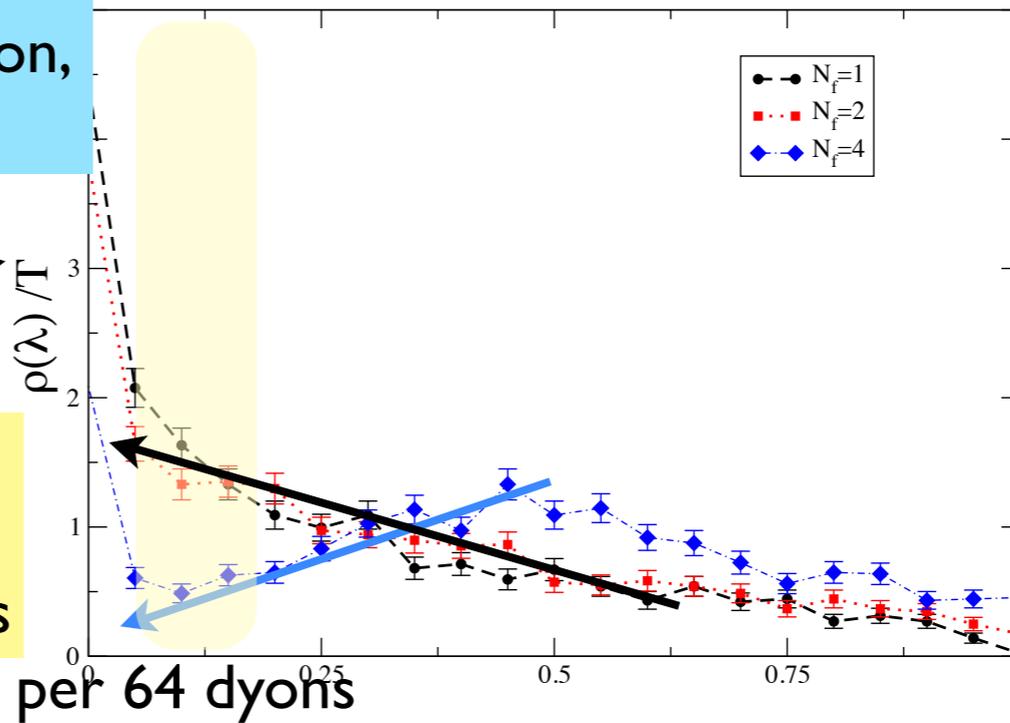
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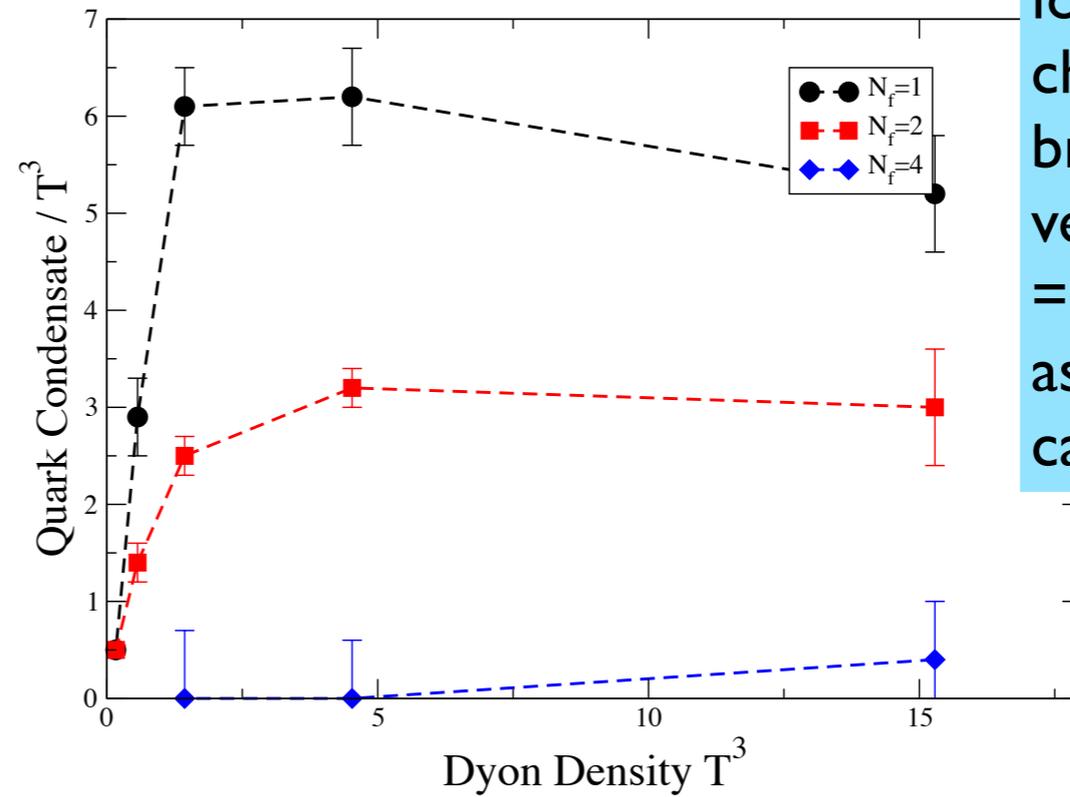
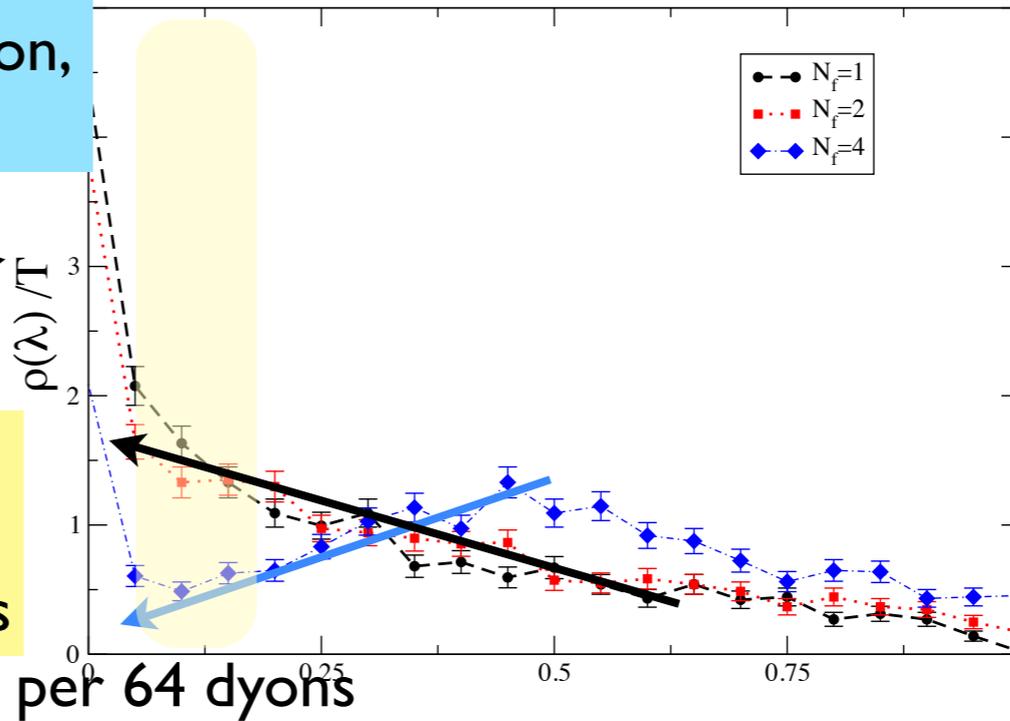
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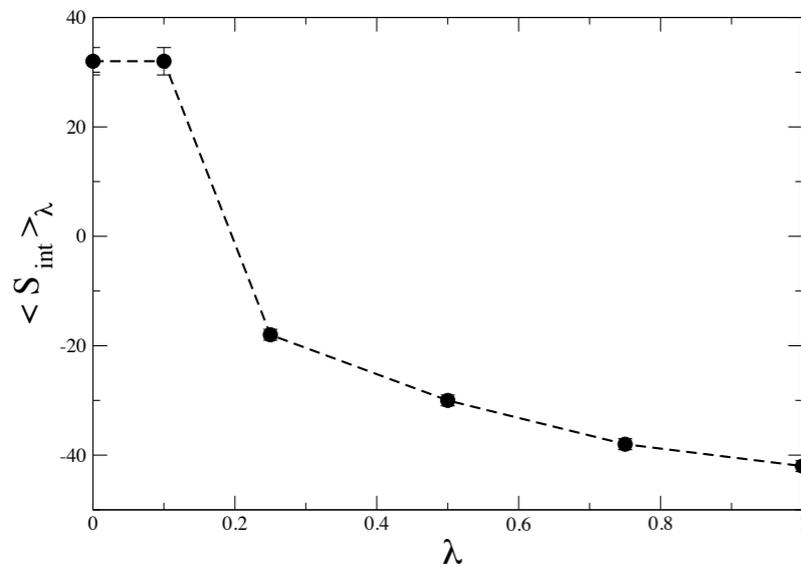
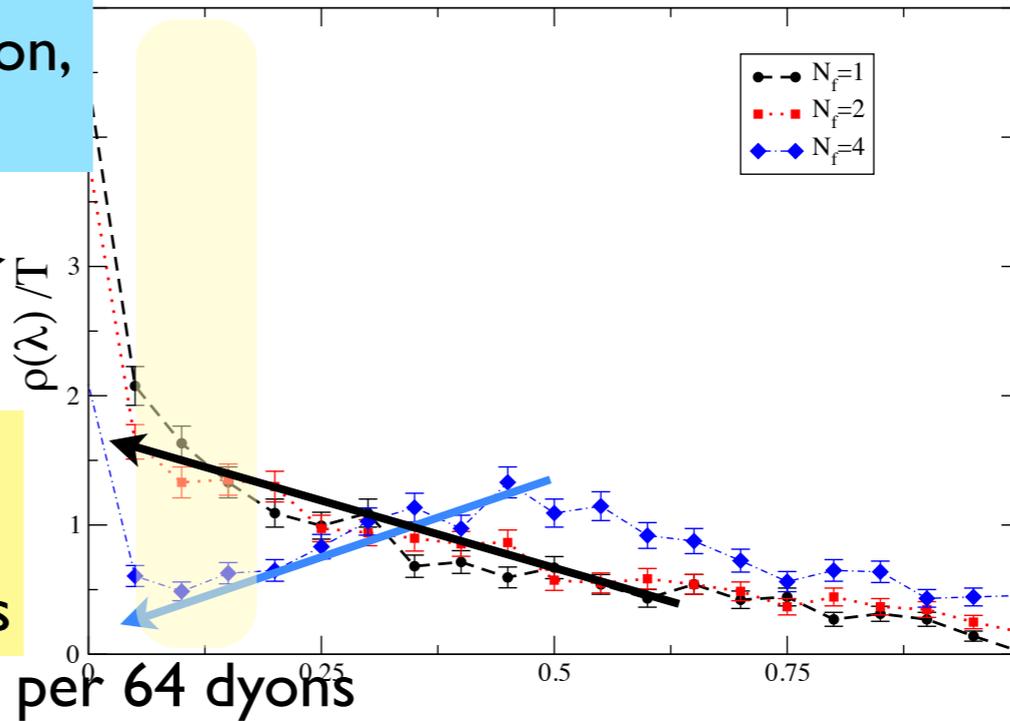
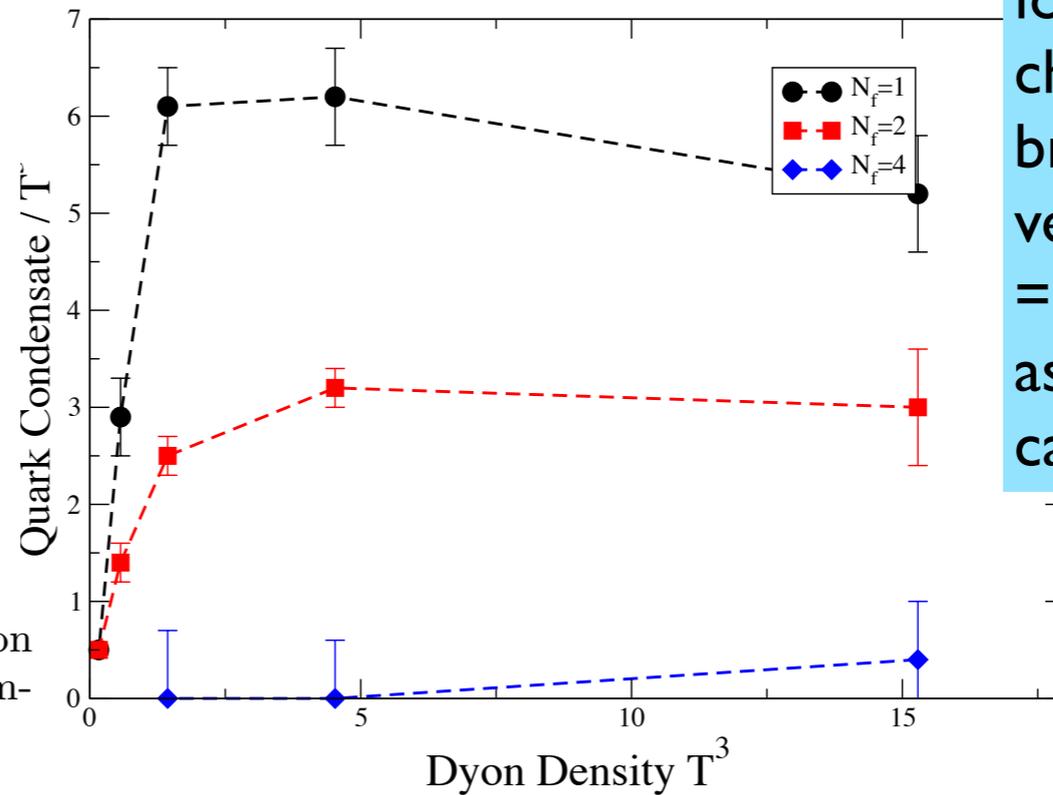


FIG. 7: The dependence of the average interaction free energy on the dimensionless adiabatic parameter λ .

per 64 dyons



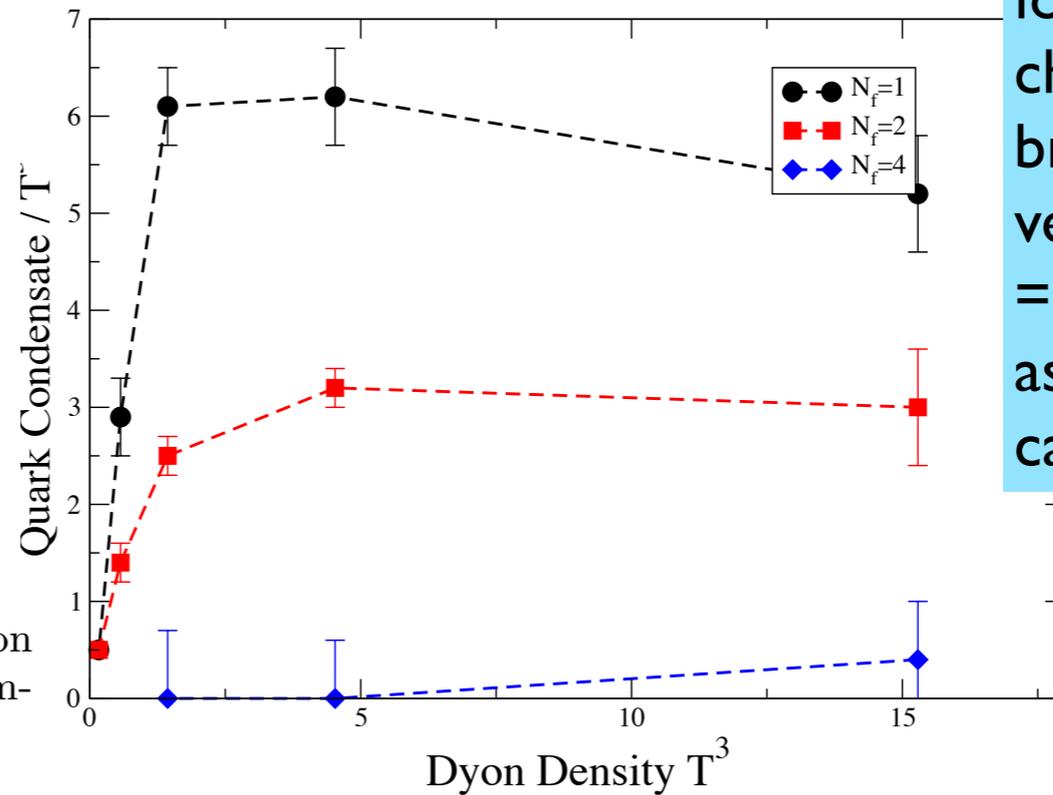
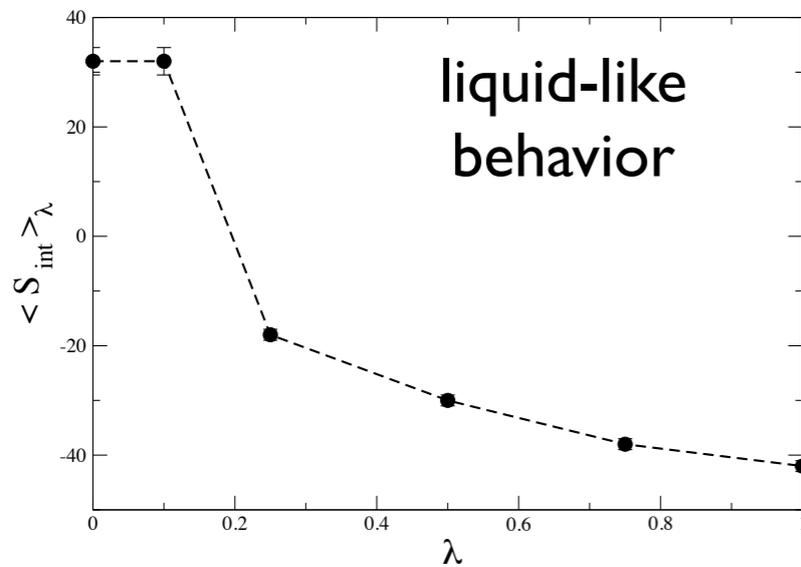
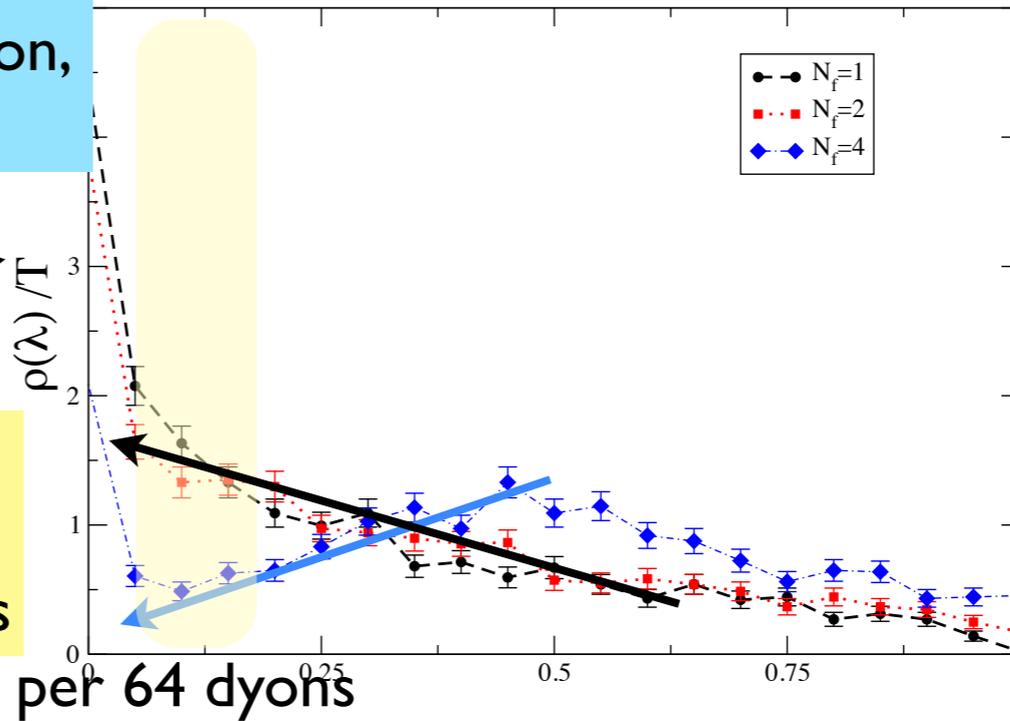
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confinement
(holonomy potential)

Holonomy potential and confinement from a simple model of the gauge topology

E. Shuryak^{1,*} and T. Sulejmanpasic^{2,†}

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Close $\bar{I}I$ pairs correspond to weak fields, which cannot be treated semiclassically and should be subtracted from the semiclassical configurations. This physical idea has been implemented in the Instanton Liquid Model via an “excluded volume”, which generates a repulsive core and stabilizes the density.

In a few important cases, in which the partition function is independently known, such subtraction can be performed exactly, *without* any parameters. The $\bar{I}I$ pair contribution to the partition function in QM instanton problem has been done via the analytic continuation in the coupling constant $g^2 \rightarrow -g^2$ by Bogomolny [14] and Zinn-Justin [15] (BZJ), who verified it via known semiclassical series. Another analytic continuation has been used by Balitsky and Yung [13] for supersymmetric quantum mechanics.

Recently Poppitz, Schäfer and Ünsal (PSU) [16, 17] used BZJ approach in the $N = 1$ Super-Yang-Mills theory on $R^3 \times S^1$, observing that the result obtained matches exactly the result derived via supersymmetry [18]. PSU papers are the most relevant for this work, as they focus on the instanton-dyons (referred to as

$v = \langle A_0 \rangle$ is Higgs VEV
shifted and rescaled,
 $v=0$ trivial limit (high T)
 $b=0$ confining ($T < T_c$)

$$\frac{1}{2} \text{Tr} P(x) = \cos \left(\frac{v(x)}{2T} \right), \quad b = \frac{4\pi^2}{g^2} \left(\frac{v}{\pi T} - 1 \right)$$

Similarly to electric holonomy
Polyakov introduced magnetic
one $\langle C_0 \rangle = \text{sigma}$

b magnetic holonomy
sigma - magnetic one

$$V_{eff} = 2n_d (-2 \cos \sigma \cosh b + n_d \mathcal{A} \cosh(2b)) .$$

$$\begin{aligned} \mathcal{M} &\sim e^{-b+i\sigma-S_d} , & \bar{\mathcal{M}} &\sim e^{-b-i\sigma-S_d} \\ \mathcal{L} &\sim e^{b-i\sigma-S_d} , & \bar{\mathcal{L}} &\sim e^{b+i\sigma-S_d} . \end{aligned}$$

4 dyon amplitudes

Poppitz+Schafer+Unsal idea:
If the quadratic term is repulsive, (+ sign)
it can lead to confinement
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$$V_{eff} = 2n_d (-2 \cos \sigma \cosh b + n_d \mathcal{A} \cosh(2b)) .$$

$$V_{pert} = \frac{\pi^2}{12} \left(1 - \frac{b^2}{S_d^2} \right)^2$$

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4 dyon amplitudes

Yet, unlike in the SUSY
 setting PSU discussed,
 there is also the perturbative
 holonomy potential to be
 overcome!

In fact the excluded volume model works well for SU(2) YM

the density is deduced from calorons
 $n(\text{dyons}) = (n(\text{calorons}))^{(1/N_c)}$
 and is large enough to make second-order in density term do its work

the only parameter A is fixed from known T_c and has a reasonable size (including the Coulomb enhancement)

electric and magnetic screening masses are even factor 2 **too large** as compared to those from lattice propagators:

their ratio m_E/m_M is well reproduced

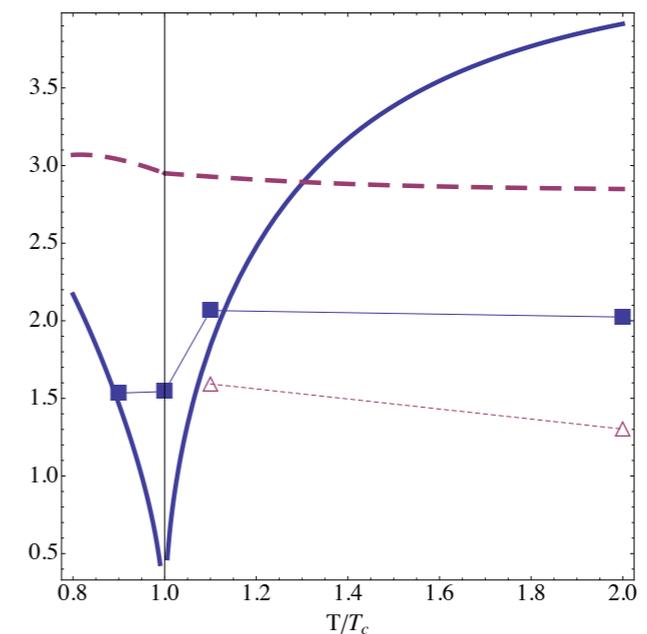
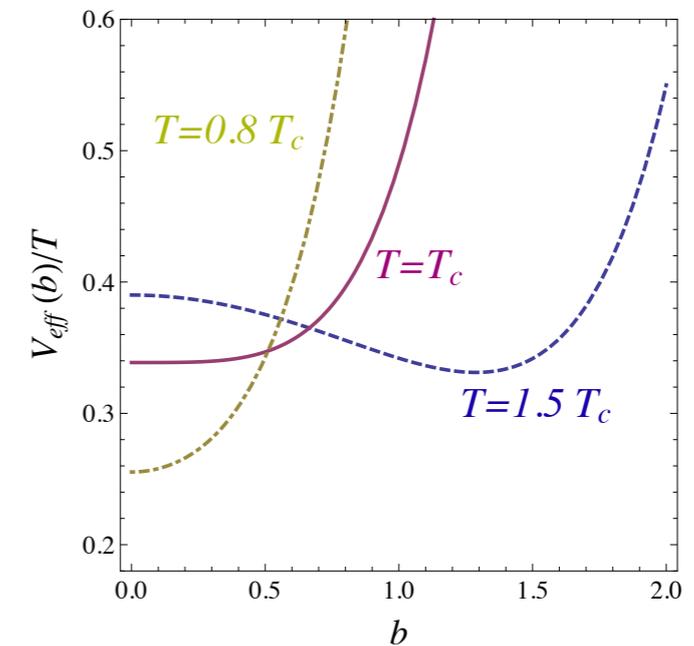


FIG. 2: The upper plot shows the effective potential $V_{eff}(b)/T$ (13) for $T/T_c = 0.8, 1, 1.5$ shown by the dashed, solid and dot-dashed lines, respectively. The plot shows electric m_E/T and magnetic m_M/T screening masses versus temperature, indicated by the solid and dashed lines, respectively. Thick lines are our model, the data points are from lattice propagators [26], the lines connecting data points are shown simply for their identification.

(going ahead of myself)

predictions: densities of the M and L dyons

crosses: “unidentified topological objects”, an upper limit

circles: identified M

L dyon size is very small and measuring $\langle P \rangle$ at its center is hard, as well as E and M charges: not done yet

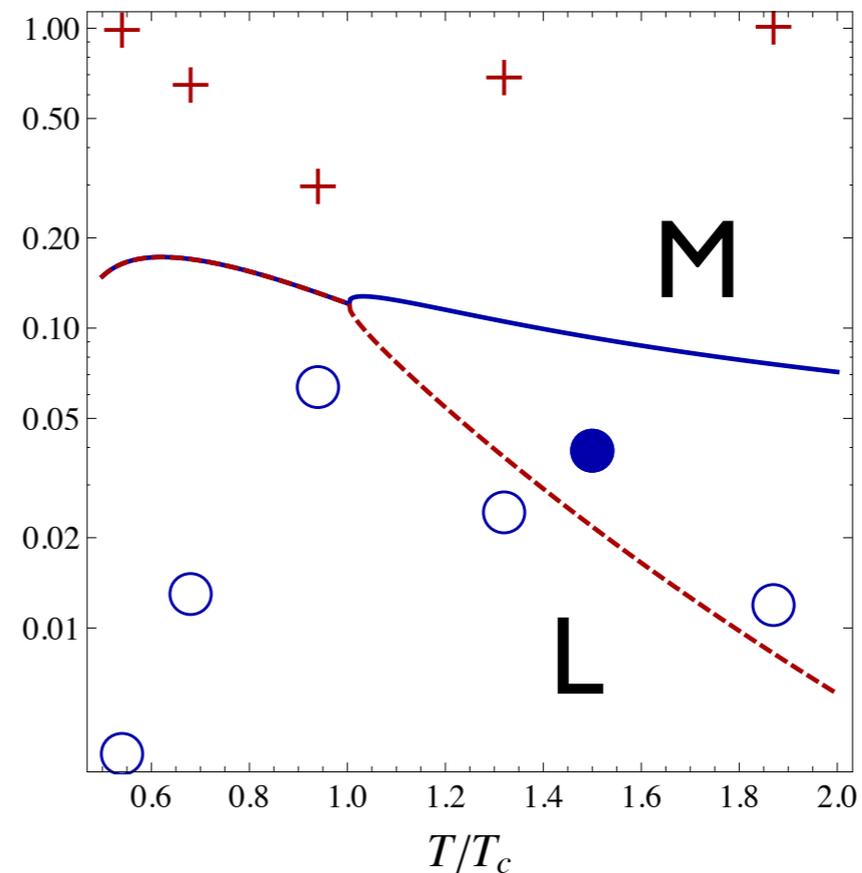
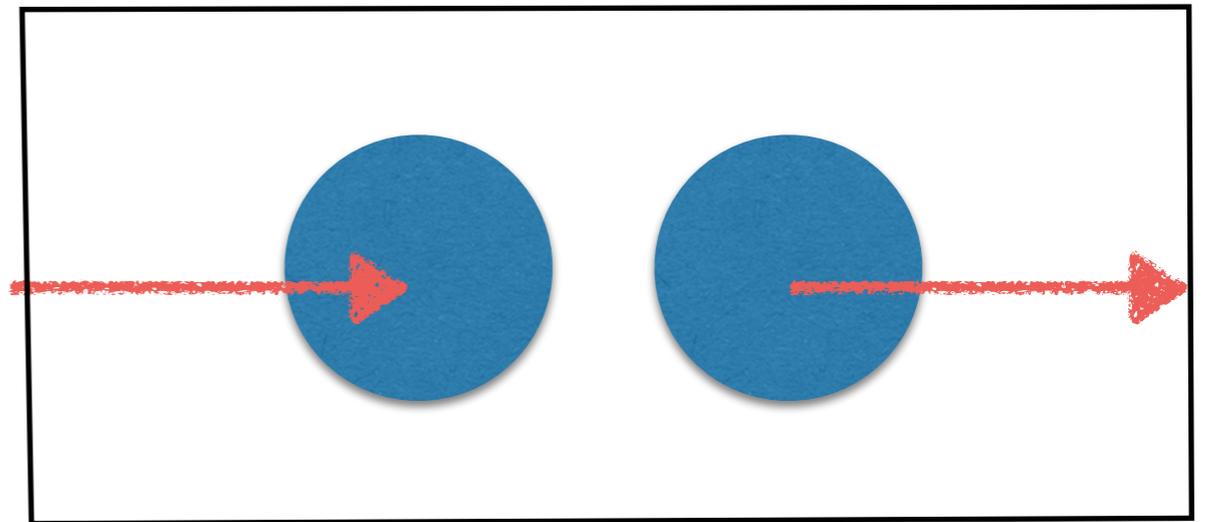


FIG. 3: Prediction of the model for the temperature dependence of the density of the instanton-dyons are shown by the lines, those with solid and dashed lines are for M, L type dyons, respectively. Open (filled) circles show identified M -type dyons from ref. [19] ([20]). The crosses show “unidentified topological objects” from [19]. Circles and crosses provide the lower and the upper bound for the dyon density.

Ongoing project:
instanton-dyon-antidyon
pairs
R.Larsen and ES



Dirac strings setting

M Mbar pair on a 3d lattice (not periodic)
start with a “combed” sum ansatz
and then action is minimized

=>”streamline configurations” found,
all the way to zero action

$e_M=0$ only Dirac string is left

$e_E=2$ massive charged gluons leave the box

Summary

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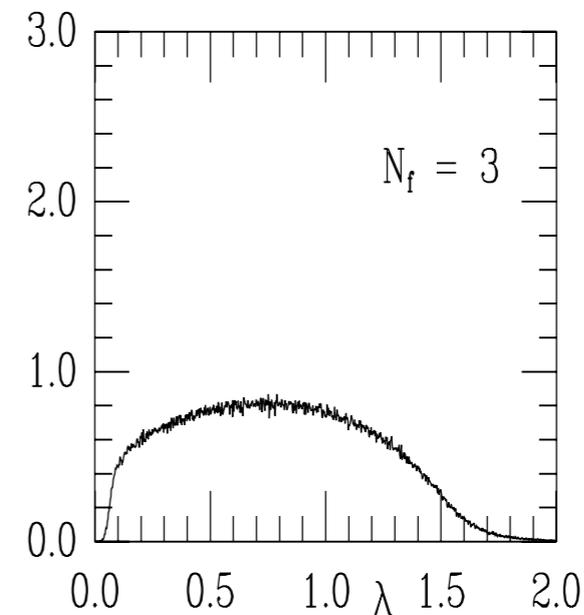
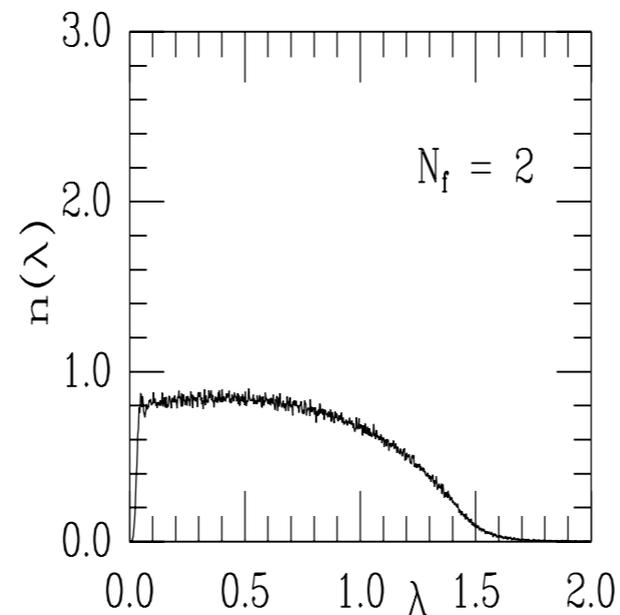
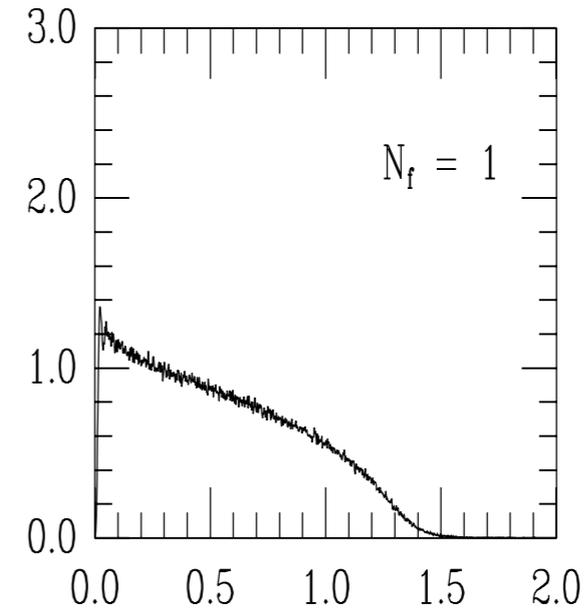
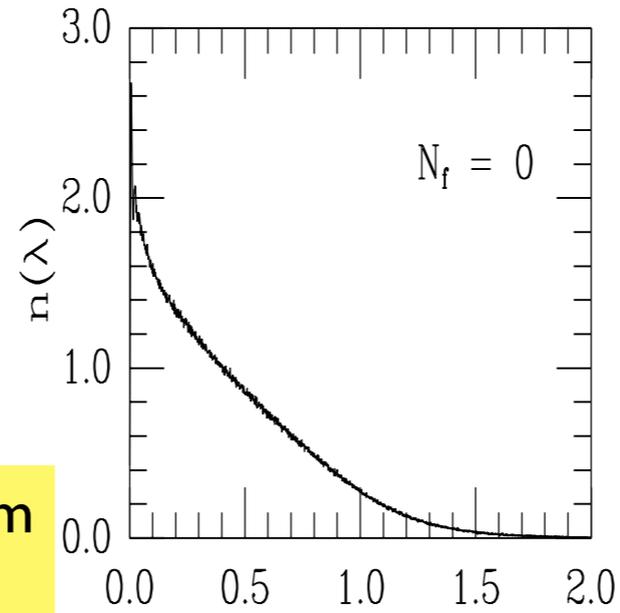
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- so we understand now why both needs large density of instanton-dyons, and why it grows with N_f (so confinement shifts to stronger coupling, lower T etc)

The spectrum of the Dirac eigenvalues

Smilga-Stern theorem
 $-|\lambda|(N_f-2)$



recently the opposite exercise was done by the Graz group

Symmetries of hadrons after unbreaking the chiral symmetry

L. Ya. Glozman,^{*} C. B. Lang,[†] and M. Schröck[‡]

Institut für Physik, FB Theoretische Physik, Universität Graz, A-8010 Graz, Austria

By eliminating ZMZ strip with width σ (about 50 modes or 10^{-4} of all) one changes masses $O(1)$: near-perfect chiral pairs are left

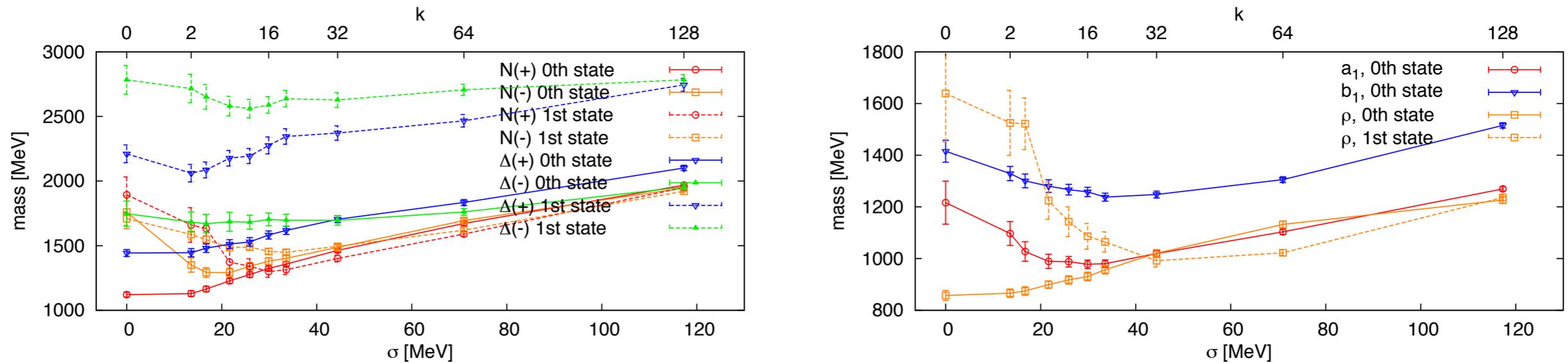


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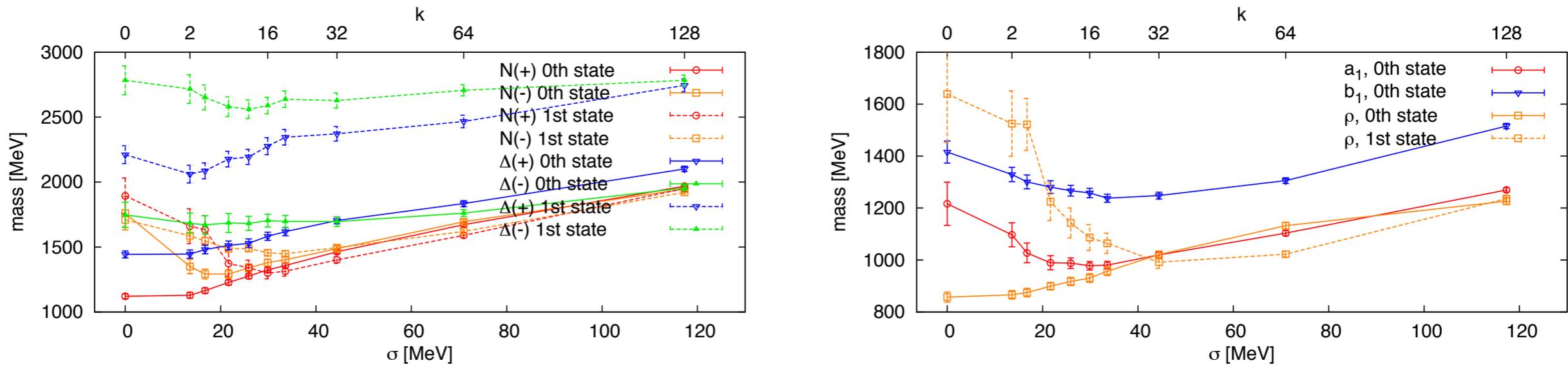


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ZMZ states are also responsible for most of the statistical noise in large scale simulations with dynamical fermions: ZMZ needs attention